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Translated by L. K.

UDC 538.4

EXACT SOLUTION OF THE PROBLEM OF AN INFINITELY CONDUCTING SPHERE WITH AN ARBITRARY VARYING RADIUS IN AN EXTERNAL MAGNETIC FIELD

PMM Vol. 36, №3, 1972, pp. 560-561

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(Received July 17, 1969)

The variation in the elctric $E = \{0,\ 0,\ E_\varphi\}$ and magnetic $H = \{H_r,\ H_\vartheta,\ 0\}$ fields caused by a motion of an infinitely conductive sphere through a homogenous magnetic field $H_0 = \{H_0 \cos\vartheta,\ -H_0 \sin\vartheta,\ 0\}$, when the radius a of the sphere varies with time in a prescribed manner, was investigated in [1, 2]. Below we consider the same problem for the case when the dependence $a = a(t)$ is arbitrary.

The problem is reduced to solution of the Maxwell equations satisfying the following initial and boundary conditions

$$H_r(r,\ \vartheta,\ 0) = H_0 \cos\vartheta, \quad H_\vartheta(r,\ \vartheta,\ 0) = -H_0 \sin\vartheta, \quad E_\varphi(r,\ \vartheta,\ 0) = 0$$
$$H_r(a(t),\ \vartheta,\ t) = 0, \quad E_\varphi(a(t),\ \vartheta,\ t) - a^{\cdot} c^{-1} H_\vartheta(a(t),\ \vartheta,\ t) = 0 \tag{1}$$

The last of these relations represents the usual electrodynamic condition [3] at the surface of an infinitely conducting sphere. For convenience, in the following, we shall replace the functions H_r, H_ϑ and E_φ by a single function $u(r,\ t)$ satisfying the wave equation and the conditions

$$u(r,\ 0) = {}^3/_2, \qquad (\partial u / \partial t)_{t=0} = 0$$
$$\left(\frac{\partial u}{\partial r}\right)_{r=a} + \frac{a^{\cdot}}{c^2}\left(\frac{\partial u}{\partial t}\right)_{r=a} + \frac{1}{c^2}\frac{d}{dt}\left[a^{\cdot} u(a,\ t)\right] + 2\frac{a^{\cdot 2}}{c^2}\frac{u(a,\ t)}{a} = 0$$

in the region $\{a(t) \leqslant r < \infty,\ t \geqslant 0\}$. This yields the following expressions for the unknown functions:

$$H_r(r,\ \vartheta,\ t) = \frac{2H_0 \cos\vartheta}{r^3} \int_{a(t)}^{r} u(x,\ t)\, x^2\, dx$$
$$H_\vartheta(r,\ \vartheta,\ t) = -H_0 u(r,\ t) \sin\vartheta + \frac{H_0 \sin\vartheta}{r^3} \int_{a(t)}^{r} u(x,\ t)\, x^2\, dx \tag{2}$$
$$E_\varphi(r,\ \vartheta,\ t) = -\frac{H_0 \sin\vartheta}{r^2} \int_{a(t)}^{r} \frac{1}{c}\frac{\partial u}{\partial t}\, x^2 dx + \frac{a^{\cdot}}{c} H_0 \sin\vartheta\, \frac{a^2}{r^2}\, u(a,\ t)$$

The lower limit of integration in (2) is chosen with the boundary conditions (1) taken into account.

The problem is solved using the method of integral transforms. We omit the intermediate calculations and quote the final result

$$u(r, t) = \frac{3}{2} - \frac{1 - 2\beta(t - r/c)}{2r} A\left(t - \frac{r}{c}\right) +$$
$$+ \frac{c}{2r} \frac{1 + \beta(t - r/c)}{A^2(t - r/c)} \int_0^{t-r/c} A^2(z) \left\{\exp - c \int_z^{t-r/c} \frac{dy}{A(y)}\right\} dz$$

here $z = t - a(t)/c$ and $A(z) = a(t(z))$.

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Translated by L. K.

UDC 539.3

ON THE CONVERGENCE OF THE METHOD OF FINITE ELEMENTS IN THE ANALYSIS OF MEMBRANE DYNAMICS

PMM Vol. 36, №3, 1972, pp. 561-565
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(Received May 18, 1971)

The rate of convergence of the method of finite elements in the analysis of natural membrane vibrations is investigated. The analysis is carried out on the basis of elements of two kinds constructed herein.

The convergence criteria of the method of finite elements are formulated in [1, 2]. Their theoretical foundation is given in [3], where it is shown that they are a sufficient condition assuring convergence in energy as the number of elements increases. An analogous proof is presented in [4] for a specific thin plate element.

The rate of convergence of the method is analyzed in [5, 6] in an example of one-dimensional systems. This question is investigated in [7] for a rectangular plate whose two opposite sides are simply supported.

1. To obtain the finite membrane element, let us use the general scheme of the method expounded in [2]. Let the membrane be divided into elements in the shape of parallelograms by line segments (Fig. 1). The points of intersection of the segments are called nodes. Let us examine an individual element with the sides a, b. Let $\xi 0 \eta$